

Model of a transverse Josephson effect driven by inhomogeneous magnetization in superconductor/ferromagnet/superconductor junctions

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We investigate transverse charge and spin dc Josephson current in superconductor/ferromagnet/superconductor junction where the ferromagnet has inhomogeneous magnetic structure. The transverse Josephson effect arises from non-trivial structure of the magnetization. The magnetic structure manifested in the transverse charge Josephson effect is essentially different from that discussed in the context of anomalous Hall effect, reflecting the dissipationless nature of Josephson current. Possible candidates of magnetic structure to verify our prediction are also discussed.

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Recently, the interplay between superconductivity and ferromagnetism has received much attention.[1–4] In particular, generation of spin-triplet pairing in ferromagnet/superconductor junction is of paramount importance.[5] Equal-spin triplet pairing emerges due to spin flip scattering in ferromagnetic multilayer or inhomogeneous ferromagnet. Spin-polarized supercurrent, carried by equal-spin triplet pairing, is a new ingredient for spintronics applications. Recent experiments have successfully demonstrated the presence of spin-triplet pairing by observing Josephson current through strong ferromagnet.[6–8] Up to now, in ferromagnetic Josephson junctions, longitudinal Josephson current has been investigated.[9–11]

The Hall effect in ferromagnet has been discussed intensively in the context of anomalous Hall effect. [12] The anomalous Hall effect arises from non-trivial spin structure, which is associated with the spin Berry phase effect. [13–17] It is shown that the Hall conductivity contains the terms stemming from non-trivial spin configurations such as vector spin chirality $\mathbf{S}_i \times \mathbf{S}_j$ [18] and scalar spin chirality $\mathbf{S}_i \cdot (\mathbf{S}_j \times \mathbf{S}_k)$ [16], where \mathbf{S}_i is localized spin with position i . Motivated by these studies, in this paper, we consider transverse Josephson effect driven by non-trivial magnetic structure under phase gradient. Since phase is odd in time-reversal, the magnetic structure manifested in transverse Josephson effect becomes essentially different from that in the anomalous Hall effect.

In this paper, we study transverse charge and spin dc Josephson current in superconductor/ferromagnet/superconductor junction where the ferromagnet has inhomogeneous magnetic structure. Analytic expressions of the transverse Josephson currents are obtained based on perturbative calculation. The transverse Josephson effect arises from non-trivial structure of the magnetization. The magnetic structure manifested in the transverse charge Josephson effect is essentially different from that discussed in the context of anomalous Hall effect, reflecting the dissipationless nature of Josephson current. Possible candidates of

magnetic structure to verify our prediction are discussed.

We consider a superconductor/ferromagnet/superconductor junction. The Hamiltonian of the superconductor and the ferromagnet are given by $H_S = H_0 + H_\Delta$ and $H_F = H_0 + H_{ex} + H_\varphi$, respectively. The H_0 , H_Δ and H_{ex} represent the kinetic energy, the superconducting order, and the exchange interaction between the conducting electron and the local spins, respectively:

$$H_0 = \sum_{\mathbf{k}} \phi_{\mathbf{k}}^\dagger \xi \tau_3 \phi_{\mathbf{k}}, \quad (1)$$

$$H_\Delta = \sum_{\mathbf{k}} \phi_{\mathbf{k}}^\dagger \Delta \tau_2 \phi_{\mathbf{k}}, \quad (2)$$

$$H_{ex} = -J \sum_{\mathbf{k}, \mathbf{q}} (\phi_{\mathbf{k}-\mathbf{q}}^\dagger \boldsymbol{\sigma} \phi_{\mathbf{k}}) \cdot \mathbf{n}_{\mathbf{q}} \quad (3)$$

with $\xi = \varepsilon_{\mathbf{k}} - \varepsilon_F \equiv \frac{\hbar^2 \mathbf{k}^2}{2m} - \varepsilon_F$ and $\phi_{\mathbf{k}}^\dagger = (c_{\mathbf{k}\uparrow}^\dagger, c_{\mathbf{k}\downarrow}^\dagger, ic_{-\mathbf{k}\downarrow}, -ic_{-\mathbf{k}\uparrow})$ where σ and τ are Pauli matrices in spin and Nambu spaces, respectively. ε_F , Δ , J , and \mathbf{n} are the Fermi energy, the gap function, the exchange coupling, and the unit vector pointing in the direction of the local spins, respectively. The localized spins can have spatial dependence, but we consider only slowly varying case. Note that we adopt the basis in Ref.[19] such that singlet pairing is proportional to the unit matrix in spin space. We consider Josephson current induced by phase gradient. The phase gradient along j direction, $\nabla_j \varphi$, enters the Hamiltonian as

$$H_\varphi = \sum_{\mathbf{k}} \phi_{\mathbf{k}}^\dagger \frac{\hbar^2}{m} k_j \nabla_j \varphi \phi_{\mathbf{k}} \quad (4)$$

where φ is half the phase of superconducting correlation and $\nabla_j \varphi$ is assumed to be spatially constant. We will treat H_{ex} and H_φ perturbatively.

With the above Hamiltonians, the charge (j_c) and spin

(j_s) currents operators in i -direction read

$$j_{c,i} = -\frac{e\hbar}{m}k_i - \delta_{ij}\frac{e\hbar}{m}\nabla_j\varphi\tau_3, \quad (5)$$

$$j_{s,i}^\alpha = \frac{\hbar^2}{2m}k_i\tau_3\sigma^\alpha + \delta_{ij}\frac{\hbar^2}{2m}\nabla_j\varphi\sigma^\alpha \quad (6)$$

where $-e$ is the electron charge and α denotes the direction of spin.

Before proceeding to the explicit calculation, let us discuss transverse Josephson currents qualitatively based on the time-reversal symmetry. [20] Consider the London equation,

$$\mathbf{j}_c = -\frac{e^2}{m}\rho \cdot \mathbf{A}. \quad (7)$$

where \mathbf{j}_c , ρ , and \mathbf{A} are, respectively, the charge current, the superfluid density, and the vector potential. Since charge current and vector potential are time-reversal odd, ρ describes the reversible and dissipationless flow of the supercurrent. Thus, the transverse current can flow without breaking time-reversal symmetry. Namely, the transverse current is allowed in even order perturbation with respect to time-reversal breaking term H_{ex} . This contrasts with the anomalous Hall effect[16] where Hall current is driven by electric field which is even under time-reversal. Thus, one can expect essentially different magnetic structure manifested in the transverse Josephson effect. Similarly, let us consider response equation of spin current,

$$\mathbf{j}_s = \frac{\hbar e}{2m}\rho' \cdot \mathbf{A}. \quad (8)$$

where \mathbf{j}_s and ρ' are spin current and superfluid density for spin current, respectively. Since spin current is even under time-reversal, ρ' relates quantities of different symmetries under time-reversal. Thus, the time-reversal symmetry should be broken to produce finite spin current. Since ρ' contains time-reversal breaking perturbation H_{ex} , this argument indicates that spin current appears only in odd order perturbation with respect to the exchange interaction.

Now, we calculate transverse Josephson currents and give their analytical expressions. We consider the unperturbed advanced Green's functions in the ferromagnet of the form $g_{\mathbf{k},\omega}^a = g_{0,\mathbf{k},\omega}^a + g_{3,\mathbf{k},\omega}^a\tau_3 + f_{\mathbf{k},\omega}^a\tau_2$ where $g_{0,\mathbf{k},\omega}^a$ and $g_{3,\mathbf{k},\omega}^a$ are normal Green's functions while $f_{\mathbf{k},\omega}^a$ is anomalous Green's function. The anomalous Green's function in the ferromagnet arises due to the proximity effect. We take into account H_{ex} up to third order and H_φ as a first order perturbation. Diagrammatic representations of the transverse currents are shown in Fig. 1. We first consider transverse charge Josephson current which can

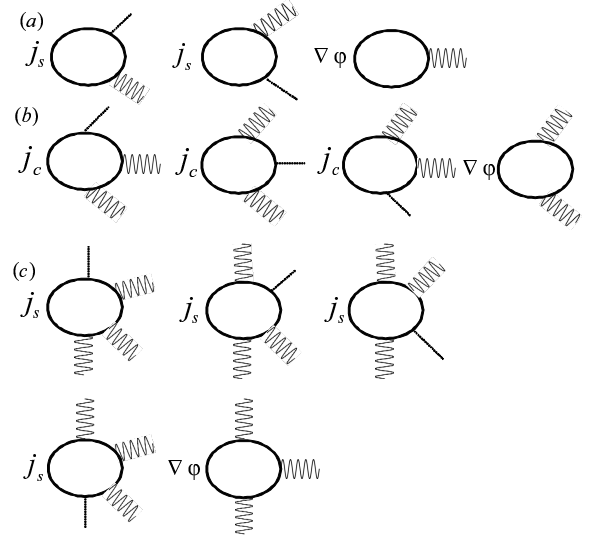


FIG. 1: Diagrammatic representations of the current densities. Diagram (a) describes first order contributions in J , (b) second-order contributions and (c) third-order contributions. The wavy lines denote the interaction with the local spin \mathbf{n} and dotted lines represent the phase gradient $\nabla\varphi$.

be represented as [21]

$$j_{c,i} = \frac{i\hbar^2 e}{mV} \sum_{\mathbf{k},\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{x}} \text{Tr} k_i G_{\mathbf{k}-\mathbf{q}/2,\mathbf{k}+\mathbf{q}/2}^<(t,t) + \delta_{ij} \frac{i\hbar^2 e}{mV} \nabla_j \varphi \sum_{\mathbf{k},\mathbf{q}} e^{-i\mathbf{q}\cdot\mathbf{x}} \text{Tr} \tau_3 G_{\mathbf{k}-\mathbf{q}/2,\mathbf{k}+\mathbf{q}/2}^<(t,t) \quad (9)$$

where V is the total volume and Tr is taken over spin and Nambu spaces. $G_{\mathbf{k}-\mathbf{q}/2,\mathbf{k}+\mathbf{q}/2}^<(t,t)$ is the lesser Green's function of the total Hamiltonian. Performing perturbation with respect to H_{ex} and H_φ , we expand the lesser component using the advanced Green's functions by the Langreth theorem.[21] Noting that $g_{\mathbf{k},\omega}^< = f_\omega [g_{\mathbf{k},\omega}^a - (g_{\mathbf{k},\omega}^a)^\dagger]$ with the lesser Green's function $g_{\mathbf{k},\omega}^<$ and the Fermi distribution function f_ω , and $\delta_{ij} = \frac{\partial k_i}{\partial k_j}$, we can compute the transverse charge Josephson current. The first order expansion in J vanishes since the Green's function is proportional to the unit matrix in spin space. Then, the leading term of the transverse charge current ($i \neq j$) is in the second order in J , which results in the form

$$j_{c,i} \cong -\frac{e\hbar}{m} J^2 \rho_c (\nabla_i \mathbf{n}(\mathbf{x}) \cdot \nabla_j \mathbf{n}(\mathbf{x})) \nabla_j \varphi \quad (10)$$

$$\rho_c = \frac{128\hbar^3}{9Vm} \sum_{\mathbf{k}, \mathbf{q}, \mathbf{q}', \omega} f_\omega \times \text{Im} \left[\varepsilon_k^2 (f_{\mathbf{k}, \omega}^a)^2 \left\{ 15(g_{0, \mathbf{k}, \omega}^a)^4 - 2(g_{0, \mathbf{k}, \omega}^a)^2 \{ 7(f_{\mathbf{k}, \omega}^a)^2 - 33(g_{3, \mathbf{k}, \omega}^a)^2 \} - \{ (f_{\mathbf{k}, \omega}^a)^2 + (g_{3, \mathbf{k}, \omega}^a)^2 \}^2 \right\} + 12\varepsilon_k (f_{\mathbf{k}, \omega}^a)^2 (g_{0, \mathbf{k}, \omega}^a)^2 g_{3, \mathbf{k}, \omega}^a \right] \quad (11)$$

We see that ρ_c depends only on junction parameters (namely, the unperturbed advanced Green's functions), independent of the details of the ferromagnet. If the anomalous Green's function $f_{\mathbf{k}, \omega}^a$ becomes zero, then $\rho_c = 0$ as expected. We have also found by the explicit calculation that the third order perturbation with respect to J does not contribute to the transverse current. Thus, up to the third order in J , only second order perturbation with respect to J remains finite as expected from the above argument based on the time-reversal symmetry.

Next, we will calculate transverse spin Josephson current. The spin current is calculated as

$$j_{s,i}^\alpha = -\frac{i\hbar^3}{2mV} \sum_{\mathbf{k}, \mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{x}} \text{Tr} k_i \tau_3 \sigma^\alpha G_{\mathbf{k}-\mathbf{q}/2, \mathbf{k}+\mathbf{q}/2}^<(t, t) - \delta_{ij} \frac{i\hbar^3}{2mV} \nabla_j \varphi \sum_{\mathbf{k}, \mathbf{q}} e^{-i\mathbf{q} \cdot \mathbf{x}} \text{Tr} \sigma^\alpha G_{\mathbf{k}-\mathbf{q}/2, \mathbf{k}+\mathbf{q}/2}^<(t, t). \quad (12)$$

In the first order of J , the spin current is represented as

$$j_{s,i}^\alpha = \frac{\hbar^2}{2m} J \rho_s \nabla_i \nabla_j \mathbf{n}^\alpha(\mathbf{x}) \nabla_j \varphi, \quad (13)$$

$$\rho_s = \frac{256\hbar^3}{9Vm} \sum_{\mathbf{k}, \omega} f_\omega \text{Im} \left[\varepsilon_k \left(\varepsilon_k g_{3, \mathbf{k}, \omega}^a + \frac{3}{8} \right) (f_{\mathbf{k}, \omega}^a)^2 \{ -(g_{0, \mathbf{k}, \omega}^a)^2 + (f_{\mathbf{k}, \omega}^a)^2 + (g_{3, \mathbf{k}, \omega}^a)^2 \} \right]. \quad (14)$$

It is seen that when the anomalous Green's function $f_{\mathbf{k}, \omega}^a$ becomes zero, then $\rho_s = 0$. By the explicit calculation, we also find that the second order term with respect to J vanishes, which is consistent with the above argument based on the time-reversal symmetry. The third order expansion with respect to J yields finite contribution to the transverse spin current. The detailed expression is quite complicated and hence omitted here. The transverse spin current in the third order in J has the form,

$$j_{s,i}^\alpha = J^3 [A' \nabla_i \nabla_j \mathbf{n}^\alpha(\mathbf{x}) + B' (\nabla_i \mathbf{n}(\mathbf{x}) \cdot \nabla_j \mathbf{n}(\mathbf{x})) \mathbf{n}^\alpha(\mathbf{x})] \nabla_j \varphi \quad (15)$$

wherer A' and B' depend solely on junction parameters.

Therefore, under phase gradient in x -direction, up to the third order in J , we have the transverse charge and spin Josephson currents in y -direction driven by magnetic structure of the form:

$$j_{c,y} = -\frac{e\hbar}{m} J^2 \rho_c (\partial_x \mathbf{n}(\mathbf{x}) \cdot \partial_y \mathbf{n}(\mathbf{x})) \nabla_x \varphi, \quad (16)$$

$$j_{s,y}^\alpha = \left[\left(\frac{\hbar^2}{2m} J \rho_s + J^3 A' \right) \partial_x \partial_y \mathbf{n}^\alpha(\mathbf{x}) + J^3 B' (\partial_x \mathbf{n}(\mathbf{x}) \cdot \partial_y \mathbf{n}(\mathbf{x})) \mathbf{n}^\alpha(\mathbf{x}) \right] \nabla_x \varphi. \quad (17)$$

These structures contrast with the normal Hall current in the ferromagnet: In the normal state, the Hall current

is driven by scalar spin chirality under electric field [16]

$$j_{c,y} \propto \left(\frac{\partial}{\partial x} \mathbf{n}(\mathbf{x}) \times \frac{\partial}{\partial y} \mathbf{n}(\mathbf{x}) \right) \cdot \mathbf{n}(\mathbf{x}). \quad (18)$$

Equilibrium spin current driven by inhomogeneous magnetic structure in the normal state is given by [22]

$$j_{s,y}^\alpha \propto \left(\frac{\partial}{\partial y} \mathbf{n}(\mathbf{x}) \times \mathbf{n}(\mathbf{x}) \right)^\alpha. \quad (19)$$

By comparing Eq.(16) and Eq.(17), and, Eq.(18) and Eq.(19), we find essentially different magnetic structures of transverse Josephson currents, which reflects that Josephson current flows in response to phase gradient, dissipationless nature of Josephson current.

Now, we discuss possible candidates of magnetic structure to verify our prediction. First, magnetization vector $\mathbf{n}(\mathbf{x})$ should have both x and y dependence. To observe transverse charge Josephson effect, $\partial_x \mathbf{n}(\mathbf{x})$ and $\partial_y \mathbf{n}(\mathbf{x})$, both perpendicular to $\mathbf{n}(\mathbf{x})$, should not be perpendicular to each other (see Fig. 2 (a)). One possible candidate is a spin vortex structure as shown in Fig. 2 (b) where $\mathbf{n}(\mathbf{x})$ is characterized by $\mathbf{n}(\mathbf{x}) = \frac{1}{a}(x, y, \sqrt{a^2 - x^2 - y^2})$

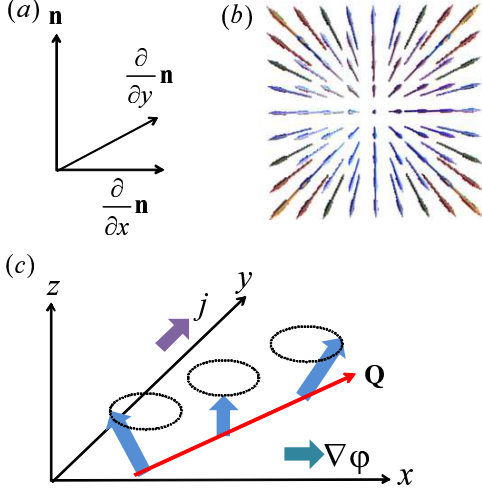


FIG. 2: (Color online) (a) Magnetization vector \mathbf{n} . (b) Vortex spin structure. (c) Conical spin structure.

with a real constant a . Then, we have

$$\frac{\partial}{\partial x}\mathbf{n}(\mathbf{x}) \cdot \frac{\partial}{\partial y}\mathbf{n}(\mathbf{x}) = \frac{xy}{a^2(a^2 - x^2 - y^2)}, \quad (20)$$

$$\frac{\partial^2}{\partial x \partial y}\mathbf{n}(\mathbf{x}) = \frac{1}{a}(0, 0, \frac{-xy}{(a^2 - x^2 - y^2)^{3/2}}). \quad (21)$$

For $xy \neq 0$, we obtain nonzero transverse Josephson currents. Conical ferromagnet, as illustrated Fig. 2 (c), is other candidate ferromagnet. The $\mathbf{n}(\mathbf{x})$ can be written as $\mathbf{n}(\mathbf{x}) = \frac{1}{\sqrt{1+b^2}}(\cos(\mathbf{Q} \cdot \mathbf{x}), \sin(\mathbf{Q} \cdot \mathbf{x}), b)$ where \mathbf{Q} is a magnetic vector and b is a real constant. Then, we have

$$\frac{\partial}{\partial x}\mathbf{n}(\mathbf{x}) \cdot \frac{\partial}{\partial y}\mathbf{n}(\mathbf{x}) = \frac{Q_x Q_y}{1+b^2}, \quad (22)$$

$$\frac{\partial^2}{\partial x \partial y}\mathbf{n}(\mathbf{x}) = -Q_x Q_y \frac{1}{\sqrt{1+b^2}}(\cos(\mathbf{Q} \cdot \mathbf{x}), \sin(\mathbf{Q} \cdot \mathbf{x}), 0) \quad (23)$$

Therefore, for $Q_x Q_y \neq 0$, we obtain finite transverse Josephson currents.

Since Josephson junction composed of conical ferromagnet Ho has been fabricated,[8] our prediction could be confirmed by ferromagnetic Josephson junctions with this material in four-terminal geometry. The presence of the predicted transverse spin current could be probed experimentally by conversion into an electrical signal via the inverse spin Hall effect (by injecting the spin current into spin-orbit coupled normal metal)[23, 24].

When proximity effect is strong such that the Green's functions in the ferromagnet have the same form as those in the bulk superconductor:

$$g_{\mathbf{k},\omega}^a = \frac{\omega - i\gamma + \xi\tau_3 + \Delta\tau_2}{(\omega - i\gamma)^2 - \xi^2 - \Delta^2} \quad (24)$$

where γ is the inelastic scattering rate by impurities, the transverse charge current Eq. (16) can be reduced to

$$j_{c,y} \cong 0.06 \times \frac{64e\hbar^3}{9m^2} \frac{\nu\varepsilon_F^2}{\Delta^4} J^2 (\partial_x \mathbf{n}(\mathbf{x}) \cdot \partial_y \mathbf{n}(\mathbf{x})) \nabla_x \varphi \quad (25)$$

for $\gamma \ll \Delta$ at zero temperature where ν is the density of states at the Fermi level. Let us estimate the transverse current for conical ferromagnet using Eqs. (22) and (25). For $\varepsilon_F \sim 1$ eV, $J \sim 1$ meV, $b = 1/\tan(4\pi/9) \cong 40$, $\nabla_x \varphi \sim (100 \text{ nm})^{-1}$, $Q_x \cong Q_y \sim (1 \text{ nm})^{-1}$, $\nu \sim 0.1$ /eV/unit cell, $\Delta \sim 1$ meV, and the lattice constant ~ 5 Å, we estimate the magnitude of the current as $j_{c,y} \sim 3 \times 10^8$ A/cm².

Spin Hall effect due to Rashba type spin-orbit coupling in superconductor [25] or Josephson junctions [26] has been discussed. In this paper, we have predicted transverse Josephson effect driven by non-trivial magnetic structure, and hence our results do not rely on spin-orbit coupling. In Ref. [26], spin Hall effect is obtained by applying electric bias to the Josephson junction in order to make the current time dependent. In stark contrast, we have considered stationary Josephson effect under non-trivial magnetic structure when phase gradient is applied.

In summary, we have studied transverse charge and spin Josephson current in superconductor/ferromagnet/superconductor junction where the ferromagnet has inhomogeneous magnetization. The transverse Josephson effect arises from non-trivial structure of the magnetization. The magnetic structure manifested in the transverse charge Josephson effect is essentially different from that discussed in the context of anomalous Hall effect, reflecting the dissipationless nature of Josephson current.

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